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Stress Recovery in the Reduced Space for Parametric Reduced Models in Microelectronics

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Introduction

In the industrial supply chain, cutting-edge technologies are required to produce high precision engineered components and assemblies (see Figure 1). These high-tech systems are made of numerous complex components. In order to ensure their reliability and robustness, numerical simulations are made at various design stages. The bottleneck is, that due to the complexities of these systems, time and computational resources for performing such simulations are very high. Therefore, there is a need for reduced order models, which can accurately capture the behavior of the original complex models (see the workflow in Figure 2).

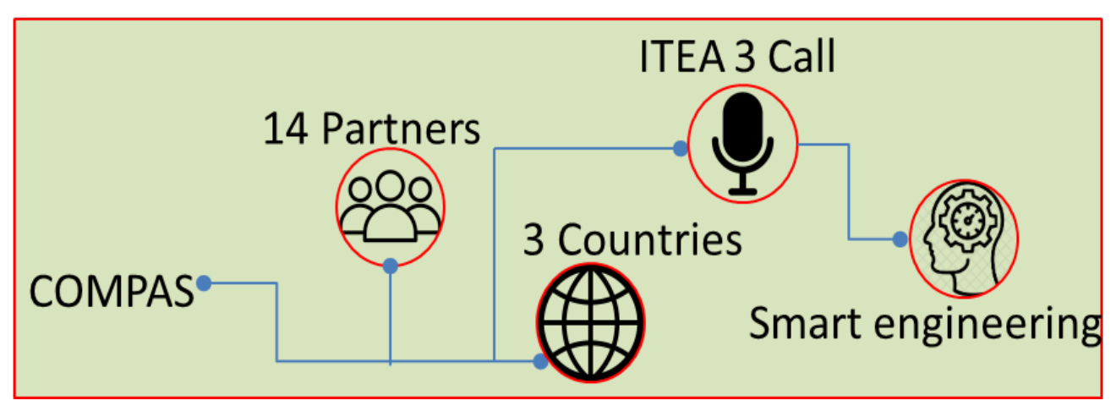

Figure. 1 Project general details. The EU project COMPAS targets smart engineering through the use of compact models for health management along the industrial supply chain.

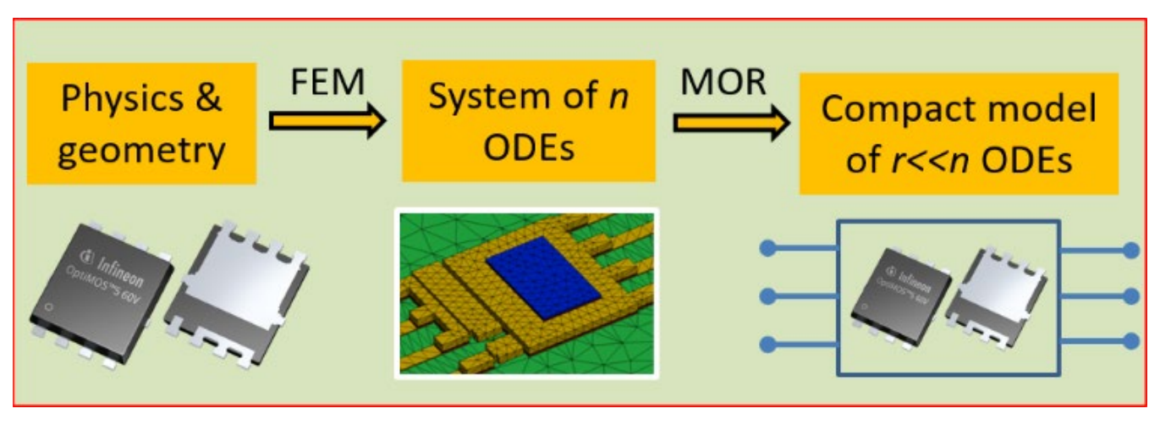

Figure. 2 Workflow for generating compact models. In the first step, the model is spatially discretised via e. g., finite element method (FEM) into a system of ordinary differential equations (ODEs) . In the second step, model order reduction (MOR) is applied to generate compact models.

Project Description

In this work, we consider a reduction of a mechanical finite element model with thermal loads and wish to observe mechanical stresses as outputs, while preserving Young's modulus as a parameter in the reduced order model. To make stresses available in the reduced space is mathematically obvious [3] [4]. However, it may not be straight-forward with commercial solvers, due to the lack of access to necessary data structures. A case study is a mechanical wafer-level chip-scale package, presented in Figure 3 left. The goal is to reduce it, while keeping the temperature-dependent Young's modulus as a parameter (see Figure. 3 right).

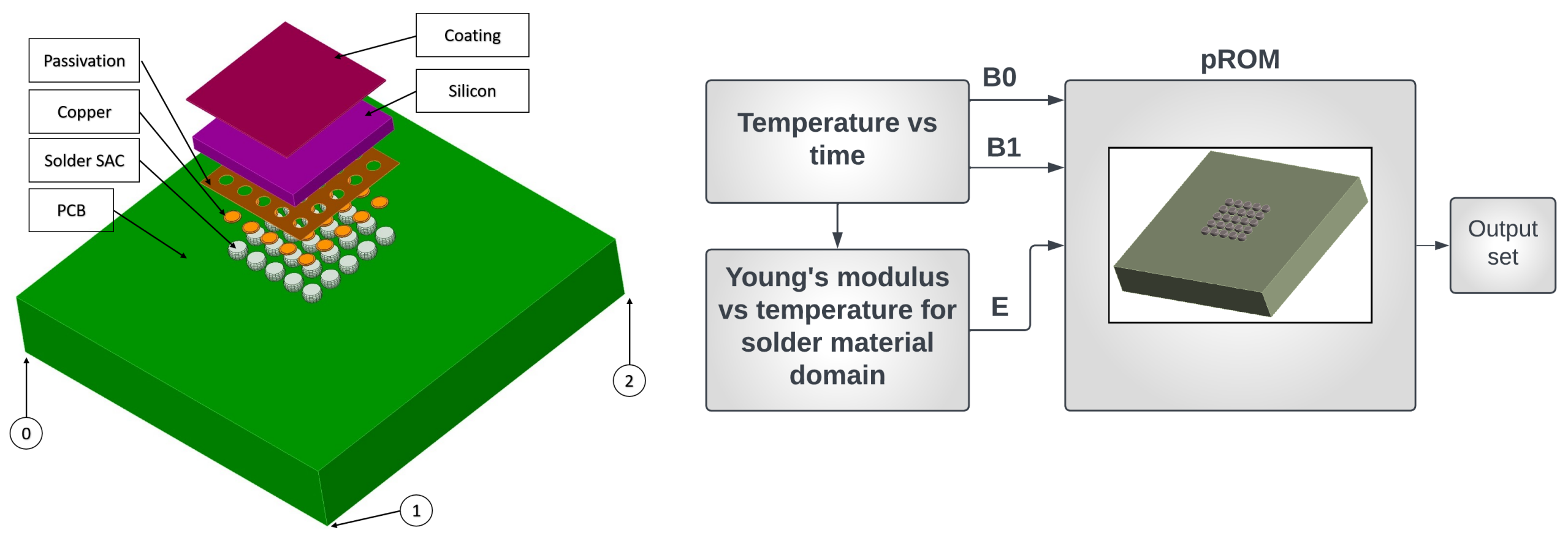

Figure.3 Left: the wafer level chip scale package is composed of six material domains. PCB is fixed at three corners from the bottom side via Dirichlet boundary conditions. Two arbitrary nodes are selected as outputs at the bottom and top surfaces of any of the corners balls of the ball grid array. Right: the system level diagram. pROM stands for parametric reduced order model.

In this model, two sources of nonlinearities can be observed. First, Young's Modulus dependency on temperature, which has a linear parametric influence in this case. It can be considered linear, because the thermal body load is applied uniformly. The second source of nonlinearity is the multilinear hardening effect in two material domains. Table 1 presents the origin of the nonlinearity of each material domain. The parametric reduced order model can be written as:

$$\Sigma_r: \begin{cases} \underbrace{V^T(K_0 + E \cdot K_1)V}_{K_r(E)} \cdot x = \underbrace{V^T(B_0 + E \cdot B_1\,)}_{B_r(E)} \cdot u(t) \\ y_r = \underbrace{CV}_{C_r} \cdot x \end{cases}$$

where $V \in \mathbb{R}^{N \times r}, K_r \in \mathbb{R}^{r \times r}, B_r \in \mathbb{R}^{r \times m}, C_r \in \mathbb{R}^{o \times r}$ and $r << N$ is the dimension of the reduced order model. Note, that m and o are the same numbers of inputs and user-defined outputs, as in the original system. $x \in \mathbb{R}^r$ is the reduced state vector and E is the Young's modulus of the specified material domain, which is now preserved as a parameter in the reduced space and can be changed within the system-level simulation.

Table 1. Material Domains with different sources of nonlinearities..

Material Domain	Source of nonlinearity	Material Domain	Source of nonlinearity
Coating	Young's modulus dependancy on temperature	Copper	Multilinear isotropic hardening
Silicon Chip	Coefficient of thermal expansion dependancy on temperature	Copper	Multilinear isotropic hardening
Passivation	Young's modulus dependancy on temperature	Copper	Young's modulus dependancy on temperature

Numerical Performance Study

The von Mises stress of the parametric reduced model matches the stress of the full scale finite element mode excellently (see Figures 9 and 10), while keeping Young's modulus as a temperature dependent parameter preserved in the symbolic form. The CPU time is reduced by a factor of 10.000 (see Table 2).

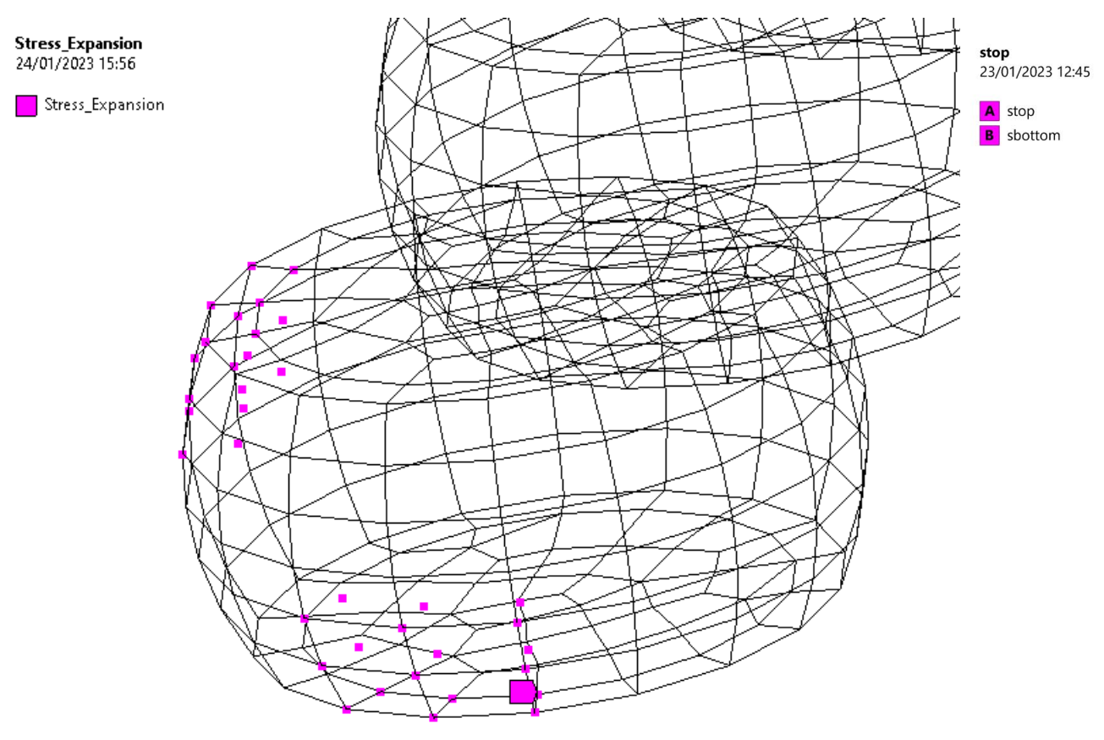

Figure. 4 The Nodes used to generate the stress output matrix through an expansion pass.

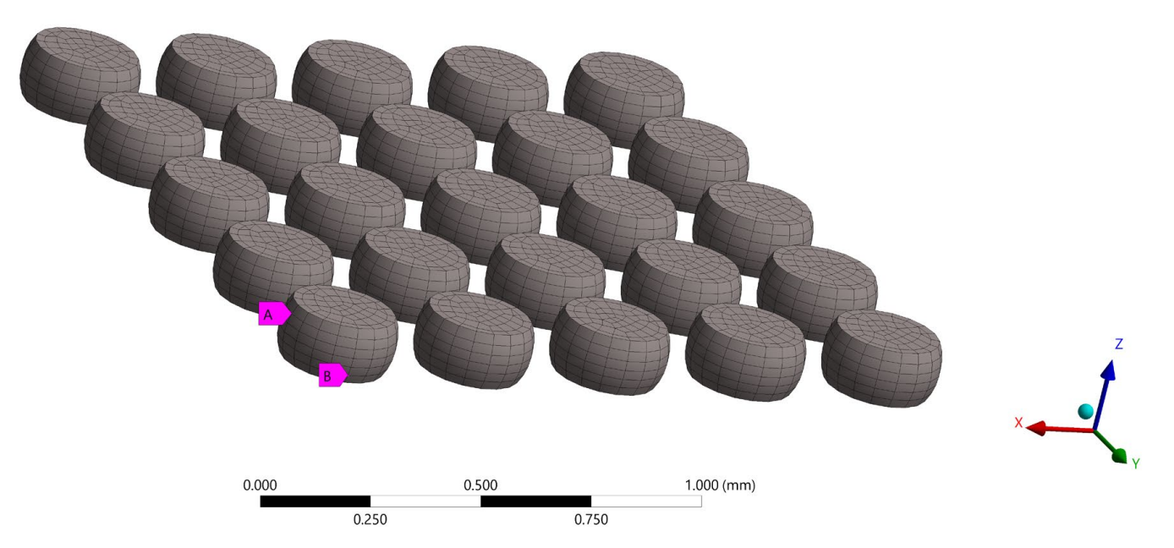

Figure. 5. Solder ball array with the output nodes highlighted.

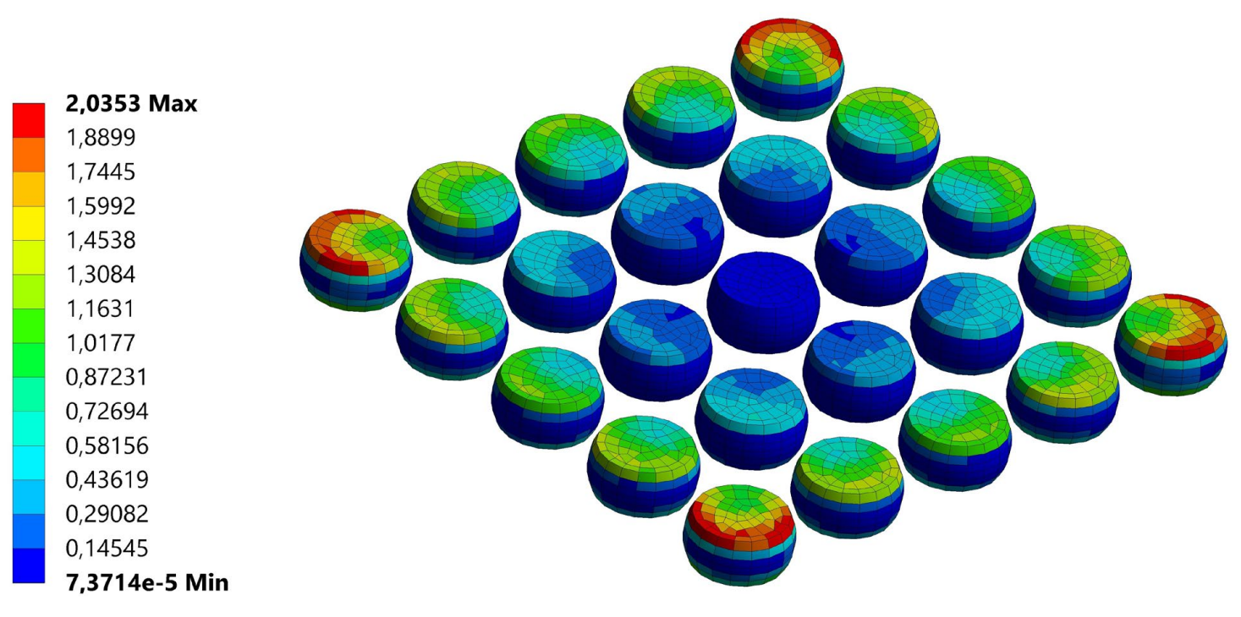

Figure. 7 plastic strain energy density distribution in the ball grid array (BGA at the last load step.)

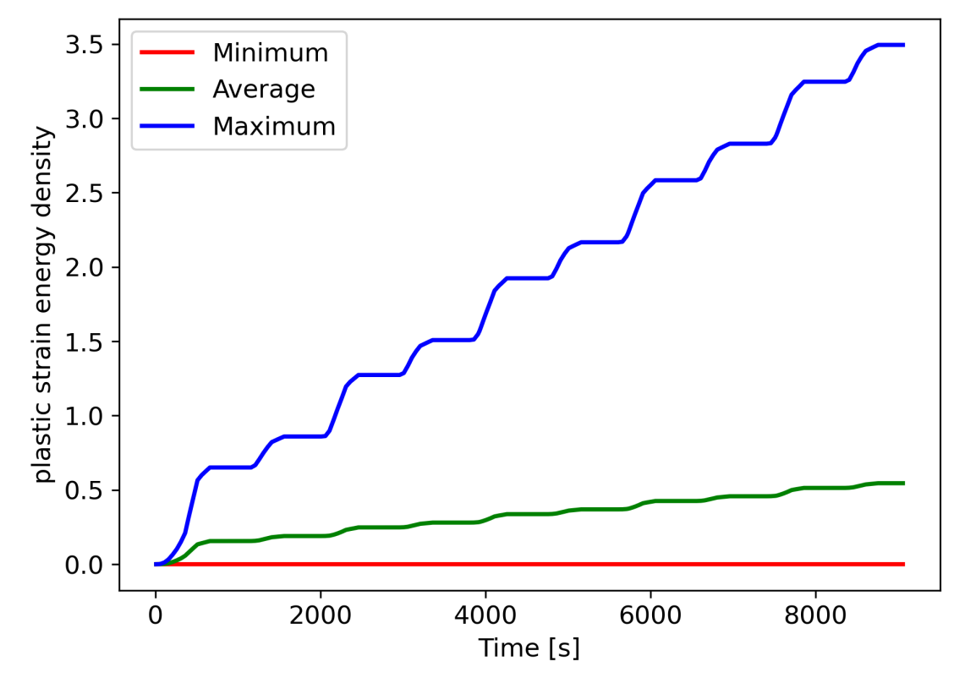

Figure. 8. Time history of the maximum, minimum, and average of the plastic strain energy density in the domain of the BGA.

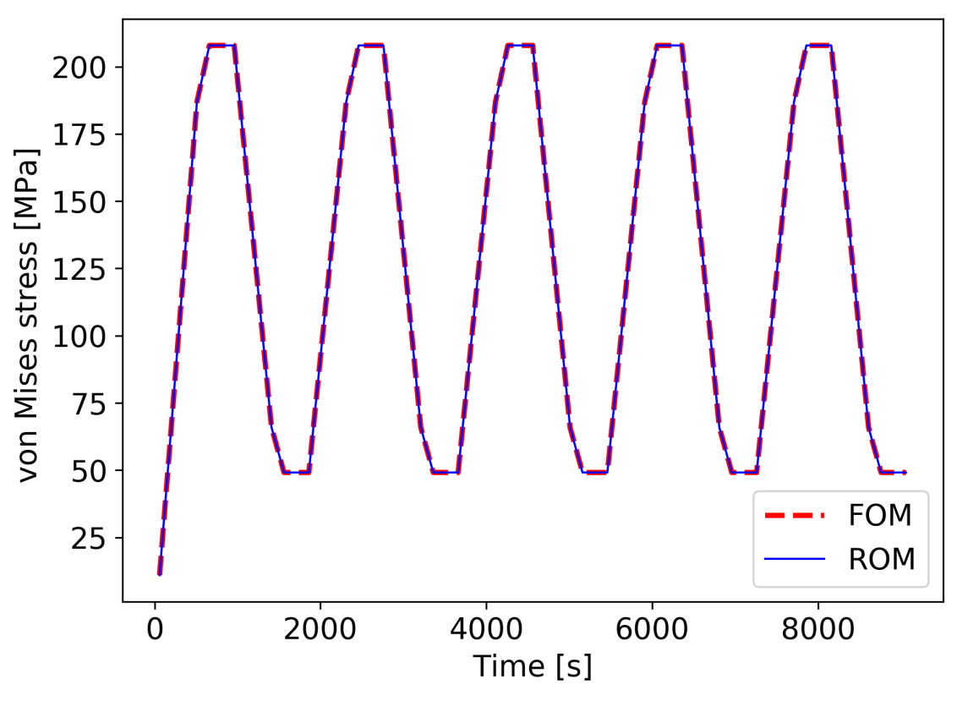

Figure. 9 Parametric reduced order model Von Mises stresses vs full order model results. As we can see a full match between them.

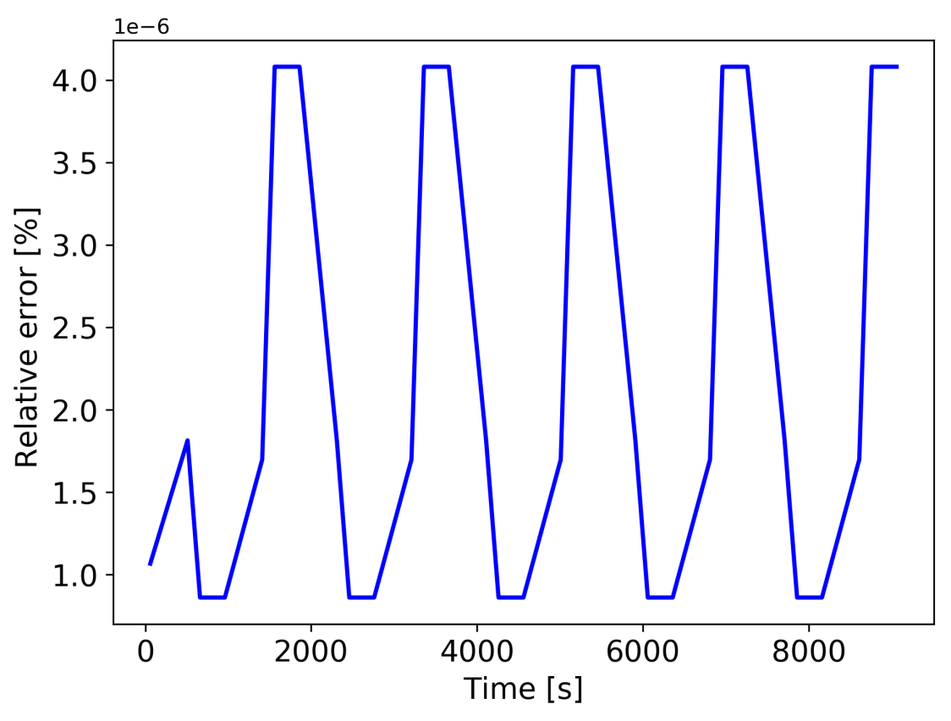

Figure. 10 Relative error of the parametric reduced order model.

Table 2. Computational time comparison between the full order model (FOM) and the parametric reduced order model (pROM). (Processor @3.0 GHz Intel Broadwell). This shows a reduction in the computational cost by a factor of 10000.

Model	DOF	Time[s]
Finite element model (FOM)	252690	28 mins
pROM generation (offline)	86	97 s
pROM simulation (online)	86	0.1600

References

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