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Algebraic Parameterization and Parametric Model Order Reduction of a Miniaturized Thermoelectric Generator

INTRODUCTION AND MOTIVATION

Electrically active implants for regenerative therapies (e.g., regeneration of bone tissue or deep brain stimulation for the treatment of motion disorders) are gaining importance within aging populations in European countries. To overcome the drawback of the battery-powered implants, a miniaturized thermoelectric generator (TEG) is designed to harvest the thermal energy from the human body and transform it into electrical energy through the Seebeck effect (see Fig.1 and Fig.2). In this work, the height of the thermopile is considered an important parameter in the design optimization process of TEG. The parametric model order reduction (pMOR) method is utilized to generate a geometrical-parameter-independent reduced order model for efficient design optimization of TEG.

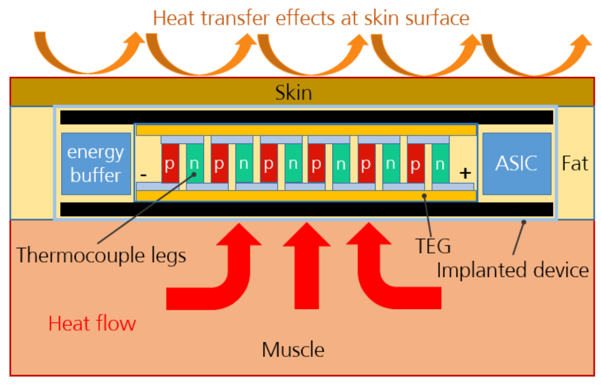

Fig. 1. TEG-powered implant inside tissue [1]

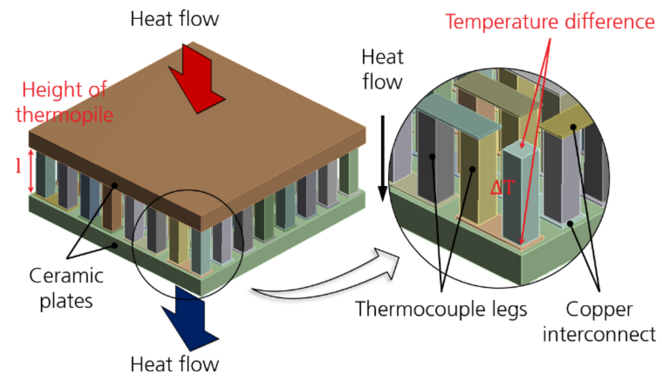

Fig. 2. Detailed schematic of a TEG [2]

METHODOLOGY

The three-dimensional model of TEG within the simplified cubic human tissue model is built in ANSYS® Workbench (2021 R2) as shown in Fig.3. We embedded a surrogate TEG in the fat layer, where the maximum temperature gradient is observed.

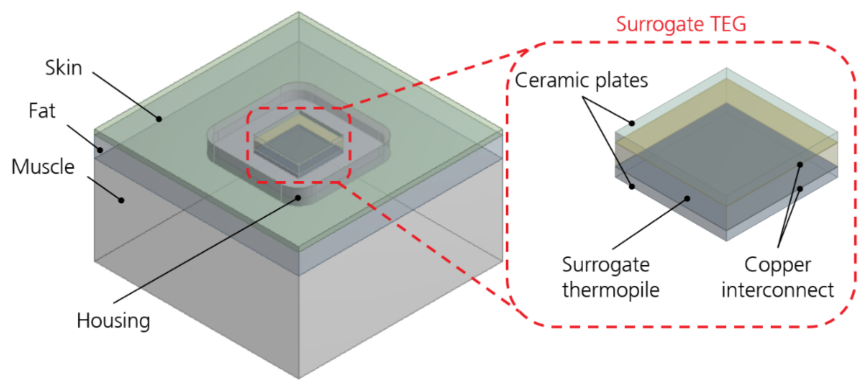

Fig. 3. Human tissue model incorporating the surrogate TEG

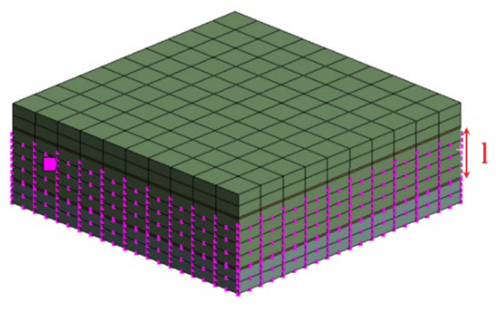

Fig. 4. Scaling the height of thermopile via shifting the nodes

To calculate the temperature difference across the TEG, a static thermal analysis is performed. Furthermore, by using the algebraic parameterization method suggested in [3], a parameterized finite element model is obtained as follows:

$$\Sigma_N \begin{cases} 0 = \underbrace{\left(\frac{1}{\alpha}\cdot A_{\frac{1}{\alpha}} + A_1 + \alpha\cdot A_\alpha\right)}_{A\left(\frac{1}{\alpha},\alpha\right)}\cdot T + B\cdot u \\ y = C\cdot T \end{cases} \quad (1),$$

where N =19,731 is the dimension of the full order model. α is the scaling factor for thermopile's height l. Parameter-independent system matrices $A_{\frac{1}{\alpha}}, A_1$ and $A_\alpha \in \mathbb{R}^{N\times N}$ are computed through the following numerical scheme:

$$\begin{bmatrix} \frac{1}{\alpha_1} & 1 & \alpha_1 \\ \frac{1}{\alpha_2} & 1 & \alpha_2 \\ \frac{1}{\alpha_3} & 1 & \alpha_3 \end{bmatrix} \begin{bmatrix} A_{\frac{1}{\alpha},i,j} \\ A_{1,i,j} \\ A_{\alpha,i,j} \end{bmatrix} = \begin{bmatrix} A_{\alpha_1,i,j} \\ A_{\alpha_2,i,j} \\ A_{\alpha_3,i,j} \end{bmatrix} \quad (2),$$

where $A_{\alpha_1}, A_{\alpha_2}, A_{\alpha_3}$ are the system matrices snapshotted at $l = \{l_1 = 3.0\text{ mm}, l_2 = 3.1\text{ mm}, l_3 = 3.2\text{ mm}\}$. $\alpha_1 = \frac{l_1}{l_1}, \alpha_2 = \frac{l_2}{l_1}, \alpha_3 = \frac{l_3}{l_1}$ are calculated scaling factors.

In ANSYS, the height of the thermopile is changed via shifting the nodes in TEG (see Fig.4). However, the Equation (2) needs to be solved N^2 times, that is, for each matrix entry, which is time-consuming. In this work, we rewrite Equation (2) as follows:

$$\begin{bmatrix} \frac{1}{\alpha_1} & 1 & \alpha_1 \\ \frac{1}{\alpha_2} & 1 & \alpha_2 \\ \frac{1}{\alpha_3} & 1 & \alpha_3 \end{bmatrix} \begin{bmatrix} A_{\frac{1}{\alpha}} \\ A_1 \\ A_\alpha \end{bmatrix} = \begin{bmatrix} A_{\alpha_1} \\ A_{\alpha_2} \\ A_{\alpha_3} \end{bmatrix} \quad (3),$$

Linear equations in (3) can be solved symbolically in MATLAB and the parameter-independent matrices can be analytically expressed as weighted sums of the snapshotted matrices

$$\begin{aligned} A_{\frac{1}{\alpha}} &= s_{11}A_{\alpha_1} + s_{12}A_{\alpha_2} + s_{13}A_{\alpha_3} \\ A_1 &= s_{21}A_{\alpha_1} + s_{22}A_{\alpha_2} + s_{23}A_{\alpha_3} \\ A_\alpha &= s_{31}A_{\alpha_1} + s_{32}A_{\alpha_2} + s_{33}A_{\alpha_3} \end{aligned} \quad (4),$$

where $s_{i,j}$, $i,j = 1,2,3$ are the coefficients calculated based on the scaling factors $\{\alpha_1, \alpha_2, \alpha_3\}$. In this way, Equation (3) needs to be solved only once and only linear combinations in Equation (4) remain to be calculated.

On basis of Equation (1), multivariate moment-matching based pMOR method suggested in [4] can be applied to generate a parametric reduced order model:

$$\Sigma_r \begin{cases} 0 = V^T A\left(\frac{1}{\alpha},\alpha\right) V\cdot z + V^T B\cdot u \\ y = CV\cdot z \end{cases} \quad (5),$$

where $V \in \mathbb{R}^{N\times r}$, r =34, is a global projection matrix obtained by merging the local projection matrices with $P = -A\left(\frac{1}{\alpha_1},\alpha_1\right)$ as follows:

$$\text{colspan}\{V_{\frac{1}{\alpha}}\} = \mathcal{K}_{r_1}\{P^{-1}A_{\frac{1}{\alpha}}, P^{-1}B\} \quad (6),$$

$$\text{colspan}\{V_\alpha\} = \mathcal{K}_{r_2}\{P^{-1}A_\alpha, P^{-1}B\} \quad (7),$$

$$\text{colspan}\{V\} = \text{colspan}\{V_{\frac{1}{\alpha}}, V_\alpha\} \quad (8).$$

NUMERICAL RESULTS

Method	Steps	Computational Time	Computational Complexity
Method from [3]	Solve Equation (2) $N\times N$ times	50943.52 s	$\mathcal{O}(27\times N^2)$, $N^2 = 389{,}312{,}361$
This work	1. Solve Equation (3) once	28.51 s	$\mathcal{O}(15\times p)$, $p = 884{,}899$ is number of nonzero elements in system matrices
	2. Calculate the coefficients $s_{i,j}$		
	3. Compute linear combination of snapshotted matrices (4)		

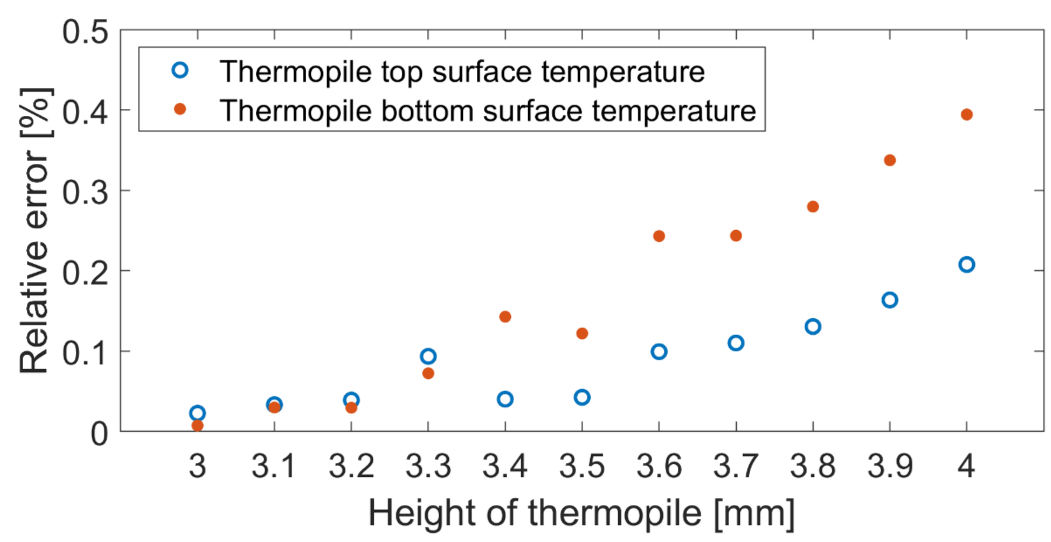

Fig. 5. Relative error in temperature results between full and reduced models

On Intel Core Processor (Broadwell, IBRS) @ 3.00 GHz, 64 GB RAM

LITERATURE

[1] C. Yuan et al., "Towards Efficient Design Optimization of a Miniaturized Thermoelectric Generator for Electrically Active Implants via Model Order Reduction and Submodeling Technique", *International Journal for Numerical Methods in Biomedical Engineering*, 36(4), 2020.

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