S. Hu[1,2], S. Stindt[1], T. Bechtold[1,2]

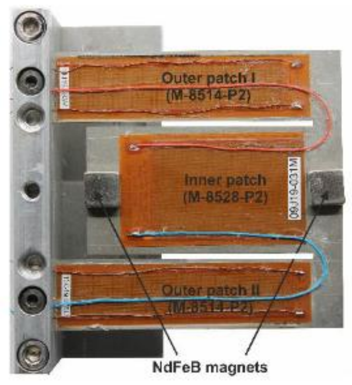
Efficient Topology Optimization of a Piezoelectric Energy Harvester

INTRODUCTION AND MOTIVATION

Energy harvesting has established itself as useful addition to batteries. This especially applies for applications involving distributed system such as wireless sensor networks. These systems often have nodes deployed in harsh environmental conditions, where batteries cannot be easily replaced. Out of all energy harvesting techniques, piezoelectric energy harvesting has proven most popular, especially due to it's high power density and scalability. Piezoelectric energy harvesters (PEH) transform mechanical vibrations into usable electrical energy. Therefore, the main design goal is having their resonance frequencies matching the excitation frequency for a given application. [1]

In this contribution, we implement the procedure introduced in [2] including modal superposition MOR, to achieve this frequency goal in an efficient way.

PLATZHALTER



PLATZHALTER

Fig. 1. An array of single beam piezoelectric energy harvester that can transform mechanical vibrations into electrical energy.

TOPOLOGY OPTIMIZATION AND MODEL ORDER REDUCTION

Topology optimization (cf. Fig. 2) makes use of the solid isotropic material model with penalization, which encodes the geometry implicitly on a fixed domain: $M_e(\rho_e) = \rho_e^{p_M} M_{e,0}, K_e(\rho_e) = \rho_e^{p_K} K_{e,0}$, where M_e, K_e are respectively the element mass and stiffness matrices with the index 0 tagging the matrices of a full material element. ρ_e is the fraction of elemental density and p_M, p_K the tunable penalty power. Standard nonlinear programming can be used to obtain the optimal material distribution on the domain. We use projection based MOR (cf. Fig. 3) to accelerate the computation of the goal function, i.e. model evaluation, in each iteration. The projection matrix V can be chosen as e.g. an eigenbasis of the matrix pencil (K, M).

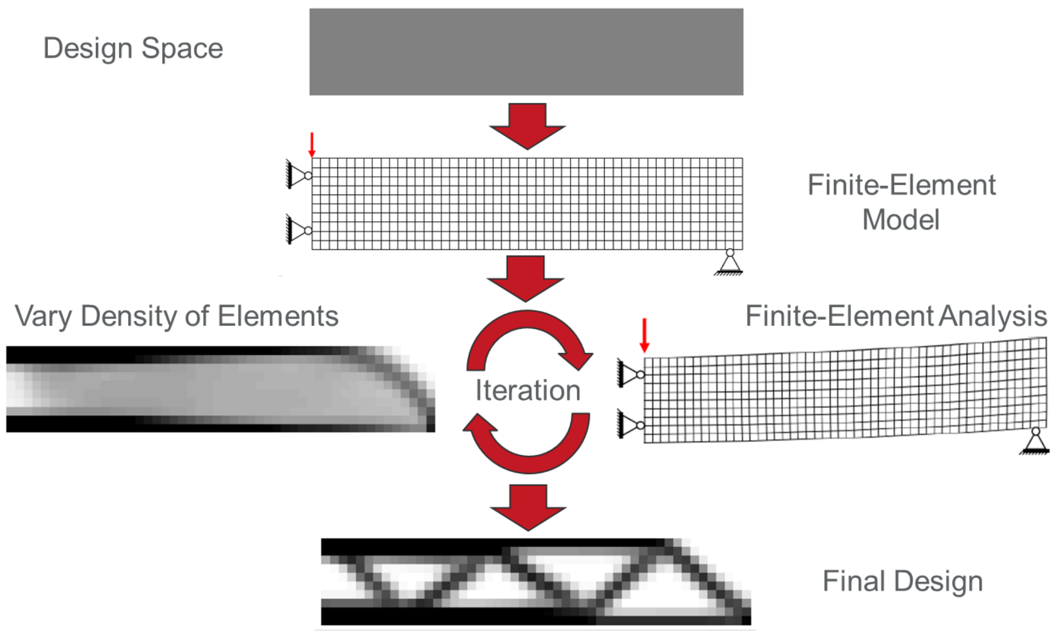


Fig. 2. Schematic diagram of SIMP based TO.

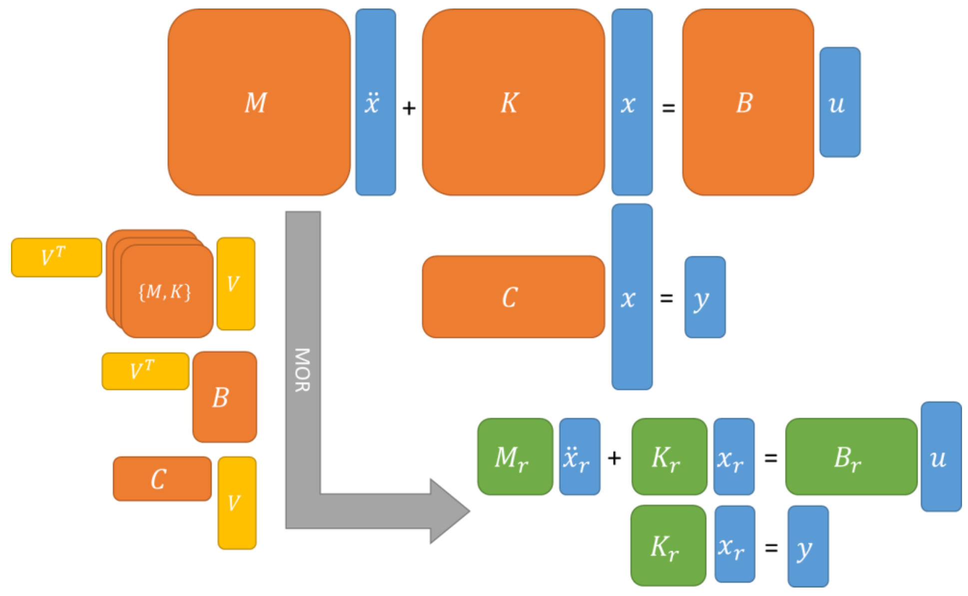


Fig. 3. Schematic diagram of projection based MOR.

NUMERICAL EXPERIMENTS

For numerical experiment, we consider a single beam PEH as shown in Fig. 4. The domain contains $80 \times 20 \times 1$ cubic finite elements

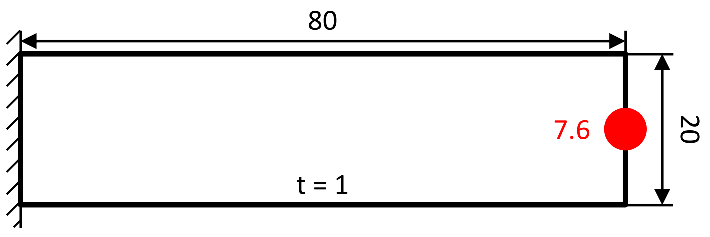


Fig. 4. Admissible design space of a single beam PEH.

The design goal is to find a structure occupying $\alpha = 50\%$ of the design space, which has its first resonance frequency at $\Omega_1 = 75$ Hz. The optimization problem defined as:

$$\min_{\rho_e} \left\{ C_d^2 = \left(\boldsymbol{x}^T (\boldsymbol{K} - \boldsymbol{\omega}^2 \boldsymbol{M}) \boldsymbol{x} \right)^2 \right\}$$

subject to:

$$\boldsymbol{\Sigma}: \begin{cases} \boldsymbol{M}(\boldsymbol{\rho_e})\ddot{\boldsymbol{x}} + \mathbf{K}(\boldsymbol{\rho_e})\boldsymbol{x} = \boldsymbol{Bu} \\ \boldsymbol{y} = \boldsymbol{Cx} \end{cases}$$

$$\sum_{e=1}^{N_e} \rho_e V_e - V^* \leq 0, (V^* = \alpha V_0),$$

$$0 \leq \rho_e \leq 1.$$

In [3], it is shown that by minimizing the dynamical compliance C_d at a frequency just below or above an eigenfrequency, this very eigenfrequency increases or decreases. Therefore, we introduced an adaptive choice of the series $\{\omega_i : \omega_i \to 75\}$:

$$\omega = \begin{cases} 0.9 \cdot 2\pi f_1, & \text{if } f_1 < 75, \\ 1.1 \cdot 2\pi f_1, & \text{if } f_1 > 75. \end{cases}$$

The elemental sensitivity is given by $\frac{d(C_d^2)}{d\rho_e} = 2C_d \cdot \boldsymbol{x}_e^T \left(p\rho_e^{p-1} \boldsymbol{K}_e - \omega^2 \boldsymbol{M}_e \right) \boldsymbol{x}_e$.

In order to reduce the computation time, modal MOR is incorporated, using the first two eigenvectors of the matrix pencil (K, M). The goal function is becomes $C_d^2 = \left(\boldsymbol{x}_r^T (\boldsymbol{\Lambda} - \omega^2) \boldsymbol{x}_r \right)^2$. For this experiment, we set $p = 5$ and the results and the corresponding convergence plot is shown in Fig. 5, where it is also compare the result without MOR. The inclusion of MOR decreased the computation time by several magnitudes (on Intel® CORE™i9-9900X CPU@ 3.5 GHz and 64 GB RAM, excluding interface time).

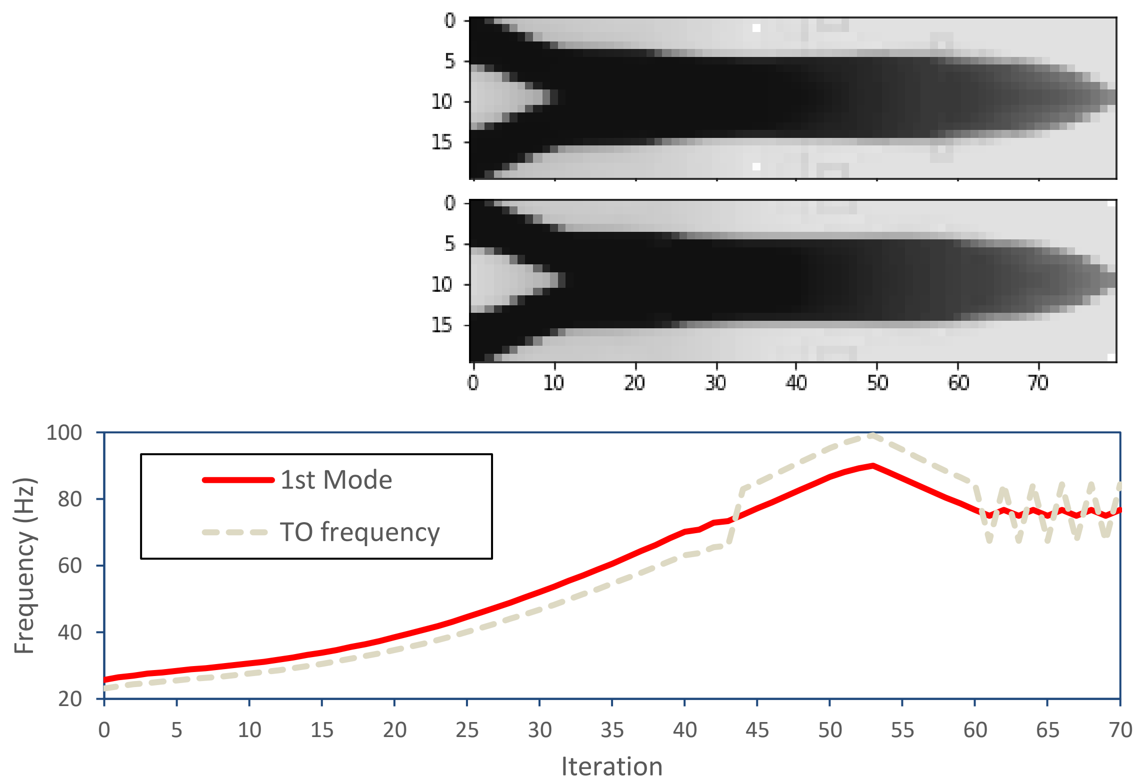


Fig. 5. Final structure (top: full vs bottom: reduced) and convergence plot of the first resonance frequency.

CONCLUSIONS AND OUTLOOK

- Our fast TO approach for resonating structures based on the GIF method and modal MOR is teste on a single beam PEH.
- Computation time is significantly reduced, while obtaining identical results.
- The algorithm will be applied to multi-resonant PEH.

LITERATURE

[1] H. Liu et al. A comprehensive review on piezoelectric energy harvesting technology: Materials, mechanisms, and applications. Applied Physics Reviews, 5(4), 041306.

[2] S. Hu et al. (2019). Fast topology optimization for resonating structures via generalized incremental frequency method and modal superposition-based model order reduction. In Advances in Structural and Multidisciplinary Optimization – Proceedings of the 13th World Congress of Structural and Multidisciplinary Optimization. Springer.

[3] N. Olhoff and J. Du (2016). Generalized incremental frequency method for topological design of continuum structures for minimum dynamic compliance subject to forced vibration at a prescribed low or high value of the excitation frequency. Structural and Multidisciplinary Optimization, 54(5), 1113–1141.

JADE UNIVERSITY OF APPLIED SCIENCES
Wilhelmshaven Oldenburg Elsfleth

[1] Department of Engineering | Jade University of Applied Sciences | Wilhelmshaven, Germany
Contact: tamara.bechtold@jade-hs.de

Universität Rostock
Traditio et Innovatio

[2] Institute of Electronic Appliances and Circuits | University of Rostock | Rostock, Germany